Important and Relevant Details to Consider When Entering a City Full of Zombies

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Abstract

Once the dead rise from their graves, and our thriving metropolises to are converted to terrible necropolises, the few remaining survivors will undoubtedly face having to re-enter cities and towns to gather necessary supplies or to finally purge the undead flesh from the streets.

In this paper, we consider the important strategic details involved with entering such a city, including: how long they can plan to safely remain in the city, how groups of people should be organized, and also strategic considerations that must be taken into account when contingency plans are drawn up.

We find a simple relationship between the rate at which zombies walk, their population densities, the rate at which individual survivors can kill zombies, and the maximal size of a population center which can safely be entered. We then demonstrate that while it might seem like a good idea for survivors who enter the city to spread out, remaining in a compact group dramatically increase survivability. Finally we consider the domain where these equations break down: the Zombie Saturation Density (ZSD), and we find a relationship between this density, the kill rate of the survivors, and the Scything Radius (SR) (the radius drawn around the group of survivors, inside of which any zombie will get killed). Knowing the SR of a given group is valuable information which can be used when contingency escape plans are drafted up.

1 The Safety Threshold Radius

Let us assume that the following data is well-known for a particular village which an individual survivor is intent on entering:

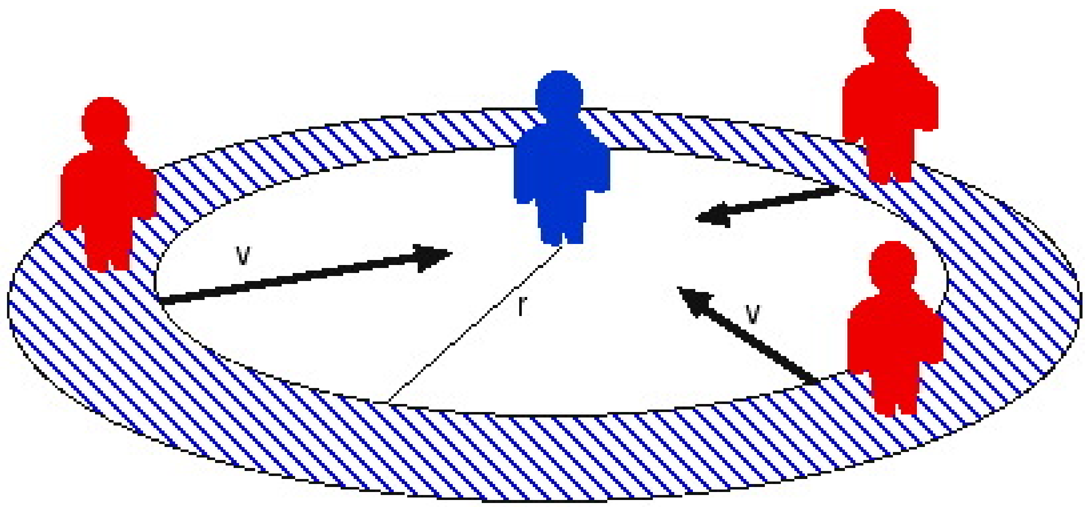
$\rho -$ average zombie density

r_{max}-radius of the town

v - average zombie walking speed

K_r-the rate at which the individual can reliably kill zombies

Then let us suppose that the survivor finds himself in the center of town when he is spotted by a zombie. The zombie will let out a call, alerting all nearby members of the walking dead who will, in turn, let out a groan summoning all other zombies within earshot. This process continues, until all of the zombies in the town are moving towards the survivors location. We assume that the zombie communication process happens instantaneously, since it is undoubtedly much faster than the average zombie walking speed v.



At what rate will zombies arrive at the survivor's location? Consider that all of the zombies in a ring of area: $\rho\delta A$, concentric around the survivor, at a radius of r. Thus, in $t = r/v$ units of time $\rho\delta A$ zombies will reach the survivor. We can alternatively describe the situation as a circle, concentric around the survivor, whose radius is increasing at a rate v. Thus, the rate at which zombies arrive at the center of the circle (K_z) can be written:

$$K_z = \frac{d}{dt}\rho A = \rho\frac{d}{dt}\pi r^2 = 2\pi\rho r v$$

In terms of units of time, the radius of the circle will be $r = vt$:

$$K_z = 2\pi\rho v^2 t\ .$$

Thus we see that the arrival rate of the zombies increase linearly in time. The survivor can kill zombies at a known rate of K_r. Thus the survivor will remain safe so long as:

$$K_r > K_z$$

or

$$K_r > 2\pi\rho v r$$

$$K_r > 2\pi\rho v^2 t \ .$$

Thus, we see that if the survivor has entered a city of sufficient size, the survivor will eventually be overwhelmed if he remains in one location for too long, since zombies will be arriving FASTER than he is able to kill them. However, if the town he has entered has a radius r_{max}: then the survivor will be able to safely deal with all the zombies in the town if:

$$K_r > 2\pi\rho v r_{max}.$$

Let us introduce a new value: the safety threshold radius r_{safe}, which can be thought of as the maximum possible town radius which the survivor has the capacity to deal with:

$$r_{safe} = \frac{K_r}{2\pi\rho v} \ .$$

If the survivor needs to enter a town whose radius $r_{max} > r_{safe}$, we advise that the survivor to plan on remaining no longer than t_{safe} units of time:

$$t_{safe} = \frac{r_{safe}}{v} \ .$$

2 Group Logistics

The consistent question in any group is whether, when approaching a town (perhaps with the goal of clearing it of undead) it is better to split the group up into individuals who spread out, or to remain in a compact group and fight together. We will show without a doubt that the survivability of such an excursion dramatically increases if the former strategy is used.

Consider the case where each the group of N individuals split up and distribute themselves into the city. What is the maximum city size in which this can safely be performed? Let us suppose that the average safety radius of the members of the group is r'_{safe}. Thus, each person can safely "suppress" an area of zombies swept out by this radius:

$$A' = (r'_{safe})^2\pi \ .$$

Thus, the total city area which can safely be "suppressed" by N members is:

$$A_{city} \leq NA'$$

$$A_{city} \leq N(r'_{safe})^2\pi$$

$$A_{city} \leq N(\frac{K_r}{2\pi\rho v})^2\pi$$

Now consider the case where the group of N individuals remain together in a compact group? The kill rate of this group (K_G) will be the summed kill rates of each of the members:

$$K_G = NK_r \ .$$

Thus, the safe radius (r''_{safe}) of such a kill-rate will be:

$$r''_{safe} = \frac{K_G}{2\pi\rho v} = \frac{NK_r}{2\pi\rho v} \ .$$

Thus, the total area of a city which can be safely entered is:

$$A_{city} \leq \pi(r''_{safe})^2$$

$$A_{city} \leq N^2(\frac{K_r}{2\pi\rho v})^2\pi \ .$$

Thus, a compact group will be able to safely manage a MUCH larger city than a group of dispersed individuals. For this reason, unless there is a specific, carefully strategically considered reason for breaking up a group when entering a city, we recommend that the group stay together under all circumstances.

3 Maximal Zombie Density and the Scything Radius

What happens in the case where the rate zombies are arriving at the survivor's location exceeds the rate at which the survivor can kill them? Will the survivor necessarily die? The answer is no. As the survivor struggles to kill as many zombies as quickly as he can, the radius at which the average zombie dies will decrease, but it will not necessarily go to zero. This is because the zombies will have a maximum density ρ_{max} as they fill the street. Once this density has been achieved in a ring around the survivor the newly arrived zombies will then pack in, at maximum density, about this radius. So long as the survivor can maintain his killing rate K_r at a steady level, a ring of constant radius will be able to be maintained around the survivor, which describes the average distance away from him at which zombies will die. We call this radius the Scything Radius (SR), evoking the survivor "mowing down" the zombies in a radius around him. The SR radius r_{SR} is determined by the maximum density ρ_{max}, the zombie speed v, and the killing rate K_r:

$$K_r = 2\pi v r_{SR}\rho_{max}$$

$$r_{SR} = \frac{K_r}{2\pi v \rho_{max}}$$

Please note that the survivability of being surrounded by zombies is DRAMATICALLY reduced if $r_{SR}$is within the average arm-length of an adult man. Survivors should try to prevent themselves from being in a situation where r_{SR} is less than a few meters. The value t_{escape}:

$$t_{escape} = \frac{r_{SR}}{v} = \frac{K_r}{2\pi v^2 \rho_{max}}$$

can be used to determine how much time a group of survivors have to cease firing and enter a helicopter, or open a manhole cover and enter the sewers, or board a vehicle. It's quite useful when making contingency escape plans, since it will rule out more complicated escape plans (climbing into a helicopter and taking off, or getting into and starting a car) given an unfortunately small K_r.

On the other hand, if sufficient planning goes into an operation, and the survivors can get their group Kill rate high enough, the scything radius will allow planners to set up camps, or ammo dumps inside the scything radius, and so provide the logistics for longer-term battles. Alternatively, we can use these formulas to determine the K_r(and thus manpower) required to actively defend a compound or camp of some given area.

4 Conclusions:

Zombies are scary and deadly, and if you need to enter a city full of them, please take our equations into account as you plan. If the city size is too large, given v,ρ,K_r: please consider the amount of time your group has before it is overwhelmed (t_{safe}), and also make some appropriate contingency escape plans consistent with your SR and t_{escape}. Only consider breaking up a group of people under very careful circumstances, since the survival rate increases dramatically the larger your group. Furthermore, if you are considering organizing the defense of a camp or fort in town, you can use the relationship between the SR and the $K_r$to plan an active defense.