The Effect of a 'Slayer' on Population Dynamics of the Undead

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Abstract

In this Paper we consider some simple models of population growth for the undead. We address the issues of short term geometric expansion, large scale growth dominated by geographic saturation, the presence of "necromancers" increasing the population at a linear rate, and the effect of an 'undead slayer' hunting the undead. The most sophisticated model demonstrates that if a 'slayer' has a capacity to hunt and kill the undead far exceeding the the abilities of the undead to hunt and kill their human prey; then the overall growth of the undead population can be surpressed, and a static population can be maintained.

1 Introduction

One ever-present question involving the study of the undead is whether the addition of a "slayer" will affect the dynamics of undead population growth. References in popular culture seem to indicate that the dilligent efforts of a "vampire slayer" or "zombie killer" can keep the undead population at bay; however, the simple fact that the undead reproduce exponentially, while a slayer can only kill at a linear rate demonstrates that the reality of the situation is, if nothing else, more complicated than depicted. This paper is a cursory exploration of the subject.

2 Simple Monotonic Population Models

The undead do not die naturally; thus, their population will increase monotonically until they have consumed all of the living humans on the earth. There are, however, numerous models for how precisely, their number will increase in time.

2.1 Many Live, Few undead, Distributed Evenly

If, for example, we assume that the number of the undead is relatively small compared to the dead; and evenly distributed, then each member of the undead community will be able to "convert" their living counterparts at a steady rate.

$$\frac{dN}{dt} = aN$$

where a is the average rate of conversion, and N is the Population. Clearly, the population in this case will increase exponentially.

This population model is overly simple, however; since the population would increase without bound, even exceeding the original living population. Furthermore, it does not take into account the possible effects of geographic saturation, or the difficulty in locating the remaining survivors to convert. Thus, the exponential model is realistic only within a well-defined regime.

2.2 Incorporating Rarity

We construct a more sophisticated population model by considering the specifics of how a particular ghoul aquires prey. Suppose that a ghoul will wander about a town of area A, and that it has the capacity to recognize a living human as prey only if it passes within a circle of area 'b' surrounding the ghoul. Then, the odds of a ghoul converting one of the $M - N$ remaining humans (where M is the original population of humans) will be

$$r(M - N)\frac{b}{A}$$

(where r is a constant which describes the odds of recognizing, catching, biting and converting the human).

Since there will be N ghouls on the prowl at any time, the rate of conversion will therefore become:

$$\frac{dN}{dt} = N(M - N)(r\frac{b}{A})$$

Note that while, for small N, the rate of incease is nearly linear; as the undead population approaches saturation (N approaches M), the rate at which the population of undead increases will figuratively 'die off'. Also note that the constant $r\frac{b}{A}$will be fairly small.

2.3 Geographic Saturation

Once a geographic location has been saturated, it cannot be expected that the density of population of undead would increase: the overall populaiton would therefore increase as a widenning circle as the undead shamble outwards to find fresh food. The radius of the circle would increase at a constant rate; related to the average velocity of a ghoul. Thus the overal population would be related to the area in the "zone of undead" :

$$N \propto \pi r^2$$

$$\frac{dN}{dt} = ar = b\sqrt{N}$$

Where a and b are relevant constants.

While this model is more accurately suited to describe the spread of the undead through the boroughs of a sprawling metropolis or suburb; a very-large-scale population model could similarly be constructed, encorporating the curvature of the earth.

2.4 Additive Term

It is not implausible that an additional term could be added to the model, accounting for a "hellmouth" or a "necromancer" type source of the undead. Presumeably the effect of such a term would be to, regardless of the circumstances, increase the population of the undead at a constant rate. The addition of a constant term to the rate of population increase is of little importance to the models discussed thus far (which will increase monotonically regardless), though we will see that it can have an important role in models which include a 'slayer' term.

3 The Effect of a 'Slayer'

The presence of a slayer among a populations of humans which are being stalked by the undead can have an effect; though as would be expected, the effect it has will depend on the undead population remaining relatively low.

3.1 Simple Slayer Model

A Slayer, working full time, would have some maximum rate at which it was able to dispatch the undead. If we assume the simplest model; within the regime where the undead are reproducing exponentially, and we additionally assume an additional "necromancer" term: the rate of increase of undead will be:

$$\frac{dN}{dt} = aN + l - k$$

where a, l, k are constants describing the natural rate of aquisition of prey of a zombie, the rate at which the undead are ARTIFICIALLY introduced to the system (via "hellmouth" or "necromancer"), and the rate at which the slayer removes undead from the system.

if $k > l$, then there will be a stationary point at $N = \frac{l-k}{a}$ where the population of the undead will be static. If the N should exceed this value, then the population of the undead will continue to increase, regardless of the Slayer's efforts. If N is below this threshold, it will decrease to nothing. Thus, we encounter two important features which a 'slayer' term can add to a model: firstly, the existance of stationary points; and second, the possibility that the population of undead could be eliminated entierly.

3.2 Incorporating Rarity

Again, a subltler population model can be generated by incorporating the effect of a rarified population of undead on the rate at which a 'slayer' can work. Supposing a 'slayer' patrols a town of area A, and that it can recognize and attempt to kill a ghoul if one enters within' an area 'c' centered on the slayer. Then the rate at which the undead changes becomes:

$$\frac{dN}{dt} = N(M-N)(r\frac{b}{A}) - N(k\frac{c}{A}) + l$$

The regime for this model is one where N relatively small since, eventually the rate at which a 'slayer' can dispatch the undead would plateau. We might expect this model to apply in its entierty to towns with small (human) populations; or alternatively we could consider it to be an approximation of the population growth in a larger city, while the numbers of the undead are still low.

In this model small N, $\frac{dN}{dt} > o$, caused by the nonzero parameter l. The significance of this term is that the population of the undead will never completely be killed off. There is a stationary point is at:

$$N_{stat} = M\frac{1}{2} - \frac{ck}{2rb} + \left[\frac{lA}{rb} + \left(\frac{ck}{2rb} - M\frac{1}{2}\right)^2\right]^{1/2}$$

The stable stationary point is an interesting feature: a stable population of the undead can be maintained through the presence of a hard-working 'slayer' on patrol.

It would be desirable for N_{stat} to be as low as possible. To this end, the effectiveness of the 'slayer' at killing the undead$'k'$, and the range in which he/she recognizes his/her prey , must be substantially larger than the equivalent two parameters for the undead (r and b respectively). Note that while this might be possible for some shambling ghouls or zombies: a vampire or warewolf slayer might have less luck at repressing population growth. Also note that unless $ck \gg rb$, that N_{stat}will be uncomfortably close to the total original population M; This indicates that the primary reason for the stabilized population of undead could be an overal scarcity of humans.

Recall that this model assumes relatively small values of N. For large enough values of N, the rate at which the 'slayer' can dispatch the undead will plateau, and the undead will begin to reproduce geometrically. This might imply that having a 'slayer' is only useful in communities with a population smaller than some threshold. Furthermore, between the upper values of N described by this model and the values of N for which the 'slayer' will be overwhelmed; there must be an UNSTABLE stationary point.

4 Conclusion

We conclude from our most sophisticated model that the existance of a 'slayer' in a community can reliably maintain the total population of undead in the community at a sustainable level if the community is sufficiently small; however our model also demonstrated that the 'slayer's ability to sense and dispatch the undead must be substantially greater than that of its prey, or else the 'sustainable level' of undead in the community will mostly be maintained by the scarceity of fresh meat.